

of the surface;  $\rho_0$ , density of the initial material;  $T_b$ ,  $T_e$ , temperature of beginning and end of binder decomposition;  $\Delta\tau_m = \Delta x_m/V$ , time necessary for the temperature in the chosen cross section of the specimen to increase from  $T_{m-1}$  to  $T_m$ ;  $\sigma_i$ , unit function;  $\bar{T}_m = (T_m + T_{m-1})/2$ ,  $i = \bar{1}, N$ ;  $m = i + 1$ ;  $\Delta T = T_m - T_{m-1}$ .

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#### IDENTIFICATION OF THE COMBUSTION FRONT OF AN OIL RESEVOIR

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UDC 536.24

A method is proposed for determining the rate of movement of the combustion zone in an oil resevoir from temperature perturbations of the earth's surface layer.

One of the most promising methods of increasing the yield of heavy and viscous fuel from oil resevoirs is the creation of a combustion zone in the resevoir. Realization of this method involves the creation of a system of igniting wells and operating systems. The combustion front, initially coincident with the surface of the igniting well, moves continuously in the direction of the operating wells at a rate of 5-15 cm a day. The form and velocity of the front depend on several factors, including the heterogeneity of the resevoir, the number and location of the operating wells, the physical properties of the resevoir, the composition, concentration, and rate of extraction of the oil, and the consumption of injected air.

It is necessary to control the combustion process in order to ensure stable resevoir combustion, an optimum shape for the combustion front, and fuller coverage of the height of the resevoir by the front. In order to examine the feasibility of using recorded temperature perturbations of the earth's surface layer caused by a combination front to determine the position of the front, the problem of heat transfer in a rock mass was examined with the following assumptions. The temperature field in the mass is described by the heat conduction equation. Before the beginning of combustion of the resevoir, the temperature field is a function of a single space coordinate  $z$ . The  $z$  axis is directed along an interior normal to the earth's surface. Boundary conditions of the third kind exist on this surface. In the subsurface layers of the earth ( $z > z^*$ ), the temperature can be assumed to be independent of time. A combustion front develops at the moment of time  $\tau = 0$  in the resevoir  $z_{in} \leq z \leq z_{ex}$ . The front is a right circular cylinder of finite length  $H = z_{ex} - z_{in}$  with its axis parallel to the  $z$  axis. The radius of the base of the cylinder is a function of  $\tau$ :  $R = R(\tau)$ . During the period of time  $0 < \tau < \tau'$ , when  $R(\tau) < s$ , the combustion zone retains the form of a circular cylinder. At  $\tau > \tau'$ , the zone takes the form of a hollow circular cylinder. The rates  $v$  of the change in the radii of the internal  $r_{in}$  and external cylinders are the same, i.e.,  $R - r_{in} = s$ . The temperature on the combustion front does not change over time.

The combustion front is identified from the values of excess temperature relative to the unperturbed temperature field in the rock mass. The excess temperature function satisfies the following system of equations:

$$c\rho \frac{\partial t(r, z, \tau)}{\partial \tau} = \frac{\partial}{\partial z} \left( \lambda \frac{\partial t(r, z, \tau)}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r\lambda \frac{\partial t(r, z, \tau)}{\partial r} \right); \quad (1)$$

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Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 56, No. 3, pp. 373-378, March, 1989. Original article submitted April 18, 1988.

$$t(r, z, 0) = 0; \quad R(0) = r_{in}(0) = 0; \quad (2)$$

$$\frac{\partial t(r^*, z, \tau)}{\partial r} = 0 \quad \text{at} \quad r^* \gg R; \quad (3)$$

$$t(r, z_{in}, \tau) = t(r, z_{ex}, \tau) = T, \quad r_{in} \leq r \leq R; \quad (4)$$

$$t(R, z, \tau) = t(z_{in}, z, \tau) = T, \quad z_{in} \leq z \leq z_{ex}; \quad (5)$$

$$\lambda \frac{\partial t(r, 0, \tau)}{\partial z} = \alpha t(r, 0, \tau); \quad (6)$$

$$t(r, z^*, \tau) = 0 \quad \text{at} \quad z^* \gg z_{ex}; \quad (7)$$

$$R = R(\tau); \quad R - r_{in} = s \quad \text{at} \quad \tau > \tau'; \quad R(\tau') = s; \quad r_{in} = 0 \quad \text{at} \quad \tau \leq \tau'. \quad (8)$$

Direct problem (1)-(8) of heat conduction in a rock mass belongs to the class of problems with movable boundaries. We solve it using a combination grid method employing explicit phase-boundary determination [1] and an explicit three-layer difference scheme [2]. The solution is obtained in a cylindrical coordinate system on a difference grid:  $z_m = z_{m-1} + h_m^z$ ;  $m = 0, 1, \dots, M$ ,  $z_0 = 0$ ,  $z_M = z^*$ ;  $r_i = r_{i-1} + h_i^r$ ,  $i = 0, 1, \dots, I^n$ ,  $r_0 = 0$ ,  $r_I = r^*$ ;  $\tau_n = n\lambda$ ,  $n = 0, 1, \dots, \ell > 1$ . The boundary conditions determine the grid functions  $t_{im}^n$ ,  $R^n$ , and  $r_{in}^n$  at the initial moment of time ( $n = 0$ ). If  $t_{im}^n$ ,  $R^n$ , and  $r_{in}^n$  are already determined for the layer  $n$ , then their values for the layer  $n + 1$  are found in the same sequence. Equations (8) are used to determine  $R^{n+1}$  and  $r_{in}^{n+1}$ . The temperature  $t_{im}^{n+1}$  at the nodal points which are at least one grid spacing distant from the movable boundaries is calculated from the difference equation

$$c\rho [(1 + \theta) \delta_\tau t_{im}^n - \theta \delta_\tau t_{im}^{n-1}] = \delta_z (\lambda_{im}^n \delta_z t_{im}^n) + \frac{1}{r_i} \delta_r (r_i \lambda_{im}^n \delta_r t_{im}^n), \quad \theta \geq 0, \quad (9)$$

where

$$\begin{aligned} \delta_\tau t_{im}^n &= \frac{1}{\lambda} (t_{im}^{n+1} - t_{im}^n); \quad \delta_z (\lambda_{im}^n \delta_z t_{im}^n) = \frac{1}{h_{m+1}^z + h_m^z} \times \\ &\times \left[ (\lambda_{i,m+1}^n + \lambda_{im}^n) \frac{t_{i,m+1}^n - t_{im}^n}{h_{m+1}^z} - (\lambda_{im}^n + \lambda_{i,m+1}^n) \frac{t_{im}^n - t_{i,m-1}^n}{h_m^z} \right]; \\ \delta_r (r_i \lambda_{im}^n \delta_r t_{im}^n) &= \frac{1}{h_{i+1}^r + h_i^r} \left[ (r_{i+1} \lambda_{i+1,m}^n + r_i \lambda_{im}^n) \times \right. \\ &\times \left. \frac{t_{i+1,m}^n - t_{im}^n}{h_{i+1}^r} - (r_i \lambda_{im}^n + r_{i-1} \lambda_{i-1,m}^n) \frac{t_{im}^n - t_{i-1,m}^n}{h_i^r} \right]. \end{aligned}$$

The necessary condition of stability of Eq. (9)

$$\frac{l\lambda [(h_i^r)^{-2} + (h_m^z)^{-2}]}{(1 + 2\theta) c\rho} \leq 0,5 \quad (10)$$

makes it possible, by varying the parameter  $\theta$ , to choose any grid spacings — as in implicit schemes. In performing the calculations, we took  $\theta = 2$ . This corresponds to a fivefold increase in  $\lambda$  compared to the maximum time step for a normal explicit scheme.

At nodal points closer than one grid spacing to the boundaries of the moving combustion zone on the layer  $n$ , the function  $t_{im}^{n+1}$  is found from an implicit difference equation. The value of  $t_{im}^{n+1}$  is determined by linear interpolation at the point located in the combustion zone at the moment  $\tau_n$  and outside the zone at the moment  $\tau_{n+1}$  [1].

The position of the external boundary of the region  $r = r^*$  is chosen on the basis of the criterion of smooth conjugation of the temperature and its derivatives up to the  $k$ -th order within the region in question and in the external space [1]. To determine the laws governing the change in the temperature field in a rock mass in the presence of a radially expanding annular zone, we conducted numerical experiments in which we took a uniform difference grid and a constant rate of displacement of the combustion zone.

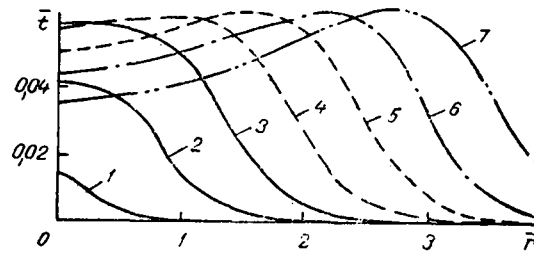


Fig. 1

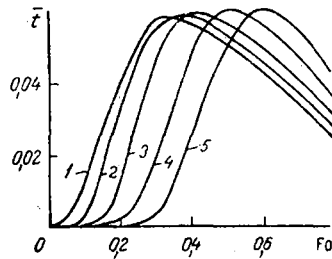


Fig. 2

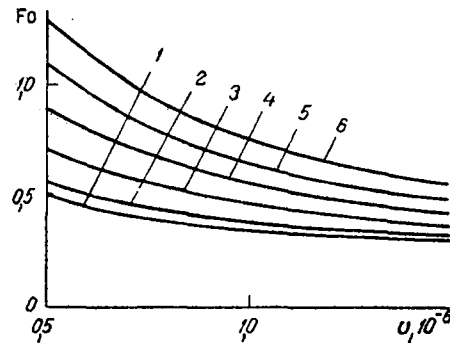


Fig. 3

Fig. 1. Radial distribution of relative temperature on the earth's surface in the vicinity of an igniting well at different moments of time: 1)  $Fo = 0.1$ ; 2) 0.2; 3) 0.3; 4) 0.4; 5) 0.5; 6) 0.6; 7) 0.7.

Fig. 2. Change in the temperature on the earth's surface in relation to the Fourier number at different distances from the igniting well: 1)  $r/z_{in} = 0$ ; 2) 0.5; 3) 1; 4) 1.5; 5) 2.

Fig. 3. Change in the Fourier numbers at which the maximum temperature is reached in relation to the velocity of the combustion front at different values of the relative distance  $r/z_{in}$ : 1)  $r/z_{in} = 0.2$ ; 2) 0.566; 3) 1.13; 4) 1.695; 5) 2.66; 6) 2.825.  $v$ , m/sec.

Figure 1 shows curves depicting the radial distribution of temperature on the earth's surface with the following initial data:  $Pe = vz_{in}/a = 5.45$ ;  $Nu = \alpha z_{in}/\lambda = 20$ ;  $\bar{s} = s/z_{in} = 0.56$ . It is evident from the figure that with a fixed value of the Fourier number  $Fo = a\tau/z_b^2$ , the curve  $t(\bar{r}, 0, Fo)$ ,  $\bar{r} = r/z_{in}$  has one maximum. At the initial moment  $0 < Fo < 0.3$ , this maximum is located at the point  $\bar{r} = 0$ . It henceforth moves in the radial direction at a velocity close to the velocity of the combustion front. Figure 2 shows graphs depicting the change in  $t(\bar{r}, 0, Fo)$  in relation to the Fourier number at different distances from the igniting well. It is evident from the figure that the rate of change in temperature  $\partial t/\partial Fo$  initially increases. Having reached an inflection point at which the temperature is about  $t_{max}/3$ , the rate then decreases. The temperature  $t(\bar{r}, 0, Fo)$  reaches the maximum value  $t_{max}$  at  $\tau_{max}$ . With a further increase in  $Fo$ , the derivative  $\partial t/\partial Fo$  at first increases monotonically and then again decreases.

Figure 3 shows curves depicting the change in the number  $Fo$  at which  $t(\bar{r}, 0, Fo)$  reaches the maximum value  $t_{max}$ . The change in  $Fo$  is shown as a function of  $v$  for different relative distances. The value  $t_{max}$  is reached earlier at an arbitrary point on the earth's surface, the greater the velocity  $v$ . Here, an increase in the distance from the well is accompanied by an increase in the derivative of the number  $Fo$  corresponding to the maximum temperature with respect to  $v$ .

Analysis of the numerical results permits the conclusion that the following measurements can be used to find the rate of displacement  $v$  of the combustion zone: a) the tem-

perature at a certain point near the earth's surface; b) the time  $\tau_{\max}$  from the moment of ignition of the reservoir to the moment when the maximum temperature  $t_{\max}$  is reached on the surface; c) the time  $t_{if}$  at which the first inflection of the function  $t$  is attained at the point on the surface.

Below, we present an algorithm for determining the rate of displacement of the combination zone in the time function by solving the inverse heat conduction problem. The algorithm is based on the method of discrete coincidence [1]. As the initial data, we assign the value of  $\tau_{\max}$  or  $\tau_{if}$  at a certain number  $J$  of points on the earth's surface or  $t(\bar{r}, 0, Fo)$  for a point with the coordinate  $\bar{r}$  with  $J$  values of the number  $Fo$ . For determinateness, we will consider the value of  $\tau_{\max j}$  ( $j = 0, 1, \dots, J - 1$ ) to be assigned. We assume that the function  $v$  can be approximated by a truncated power series

$$v = v_0 + v_1\tau + v_2\tau^2 + \dots + v_K\tau^K. \quad (11)$$

The algorithm does not undergo significant change if  $v$  is approximated by a smoothing spline or a Fourier series. If the coefficients of series (11) are determined in the usual manner, then we arrive at a direct heat-conduction problem whose solution algorithm was described above. The solution of this problem gives us model values of  $\tau_{\max(m)j}$  and the errors  $\eta_j = \tau_{\max j} - \tau_{\max(m)j}$ , which serve as a basis for subsequent refinement of the parameters  $v_j$  ( $j = 0, 1, \dots, K$ ) in (11). We use the method of successive error minimization [1] to search for optimum values of the parameters  $v_j$  that will satisfy the conditions  $|\eta_j| < \delta$  for the error  $\eta_j$ , where  $\delta$  is a certain small positive quantity. Each initial parameter  $v_k$  ( $k = 0, 1, \dots, K$ ) is correlated with a certain error  $\eta_k$  ( $k = 0, 1, \dots, K$ ). As a first approximation, we assign values of the parameters  $v_{k(1)}$ . For example, we assume that  $v_{0(1)} = \text{const} > 0$ , while  $v_{k(1)} = 0$  ( $k = 1, 2, \dots, K$ ).

Solution of the direct heat-conduction problem yields the error  $\eta_{0(1)}$  in the first approximation. We introduce a small trial step  $\Delta v_{0(1)} = v_{0(2)} - v_{0(1)}$  for the parameter  $v_0$  and we again solve the same problem. In the second approximation, we find the error  $\eta_{0(2)}$ , the error increment  $\Delta\eta_{0(1)} = \eta_{0(2)} - \eta_{0(1)}$ , and the approximate value of the derivative  $\partial\eta_0/\partial v_0 = \Delta\eta_{0(1)}/\Delta v_{0(1)}$ . Then we introduce several working steps for the parameter  $v_0$  with fixed values for the other parameters until we satisfy the condition  $|\eta_0| \leq \delta$ . The size of the working steps is determined from the formula  $\Delta v_k = \eta_k(\partial\eta_k/\partial v_k)$ . The derivative  $\partial\eta_k/\partial v_k$  can be refined after each  $\beta$ -th working step from the difference relation

$$\left(\frac{\partial v_k}{\partial \eta_k}\right)_{(\beta)} = \frac{v_{k(\beta)} - v_{k(\beta-1)}}{\eta_{k(\beta)} - \eta_{k(\beta-1)}}.$$

We will assume that for the errors  $\eta_k$  ( $k = 0, 1, \dots, b; b < K$ ), corresponding to the parameters  $v_0, v_1, \dots, v_b$ , the condition  $|\eta_k| \leq \delta$  is already satisfied. We assume further that it is necessary to determine values of the parameters  $v_0, v_1, \dots, v_{b+1}$  satisfying the inequalities  $|\eta_k| \leq \delta$  ( $k = 0, 1, \dots, b + 1$ ). First we introduce the trial step  $\Delta v_{b+1(1)}$  for the parameter  $v_{b+1}$ , i.e., we take  $v_{b+1(2)} = v_{b+1(1)} + \Delta v_{b+1(1)}$ . After each  $\beta$ -th step for the parameter  $v_{b+1}$ , we perform a cycle of calculations connected with the change in the parameters  $v_0, v_1, \dots, v_b$  until we satisfy condition  $|\eta_k| \leq \delta$  for  $k = 0, 1, \dots, b$ . In this cycle, no trial steps are introduced to find the derivative  $(\partial\eta_k/\partial v_k)_{(1)}$ . If the error of

the solution  $\Pi_{v(b+1)} = \left(\sum_{j=1}^J \eta_{j(b-1)}^2\right)^{0.5} > \Pi_{v(b)}$ ,  $J > b$ , then we take  $K = b$ . The result can be further refined by comparing other values of  $\tau_{\max j}$  from the prescribed values with the parameters  $v_k$ .

Let us present some results of the solution of the inverse heat-conduction problem in a rock mass in the case when the time  $\tau_{\max}$  over which the maximum value of temperature  $t_{\max}$  is reached at a given point of the region under consideration is measured with the error  $\Pi_\tau = (\tau_{\max} - \tau_{\max t})/\tau_{\max t}$  ( $\tau_{\max}$  and  $\tau_{\max t}$  are measured and true values of time over which  $t_{\max}$  is reached). In numerical experiments, the procedure of establishing a link between the error  $\Pi_\tau$  and  $\Pi_v$  is executed in this sequence. With assigned values for the initial data (including the rate of displacement of the combustion front  $v_t$ ), we solve the direct heat-conduction problem and we find values of  $\tau_{\max t}$ . We then solve the inverse problem, in which we assign  $\tau_{\max} = \tau_{\max t}(1 + \Pi_\tau)$  in place of  $v_t$  and we find  $v$  and  $\Pi_v = (v - v_t)/v_t$ . For values of velocity  $v$  equal to  $0.5 \cdot 10^{-6}$ ,  $10^{-6}$ , and  $1.5 \cdot 10^{-6}$  m/sec, the relative error  $\Pi_v/\Pi_\tau$  is 1.32, 1.58, and 1.92 at  $\Pi_\tau = 1\%$  and 1.3, 1.63, and 1.94 at  $\Pi_\tau = 6\%$ . At  $\Pi_\tau = 0$ ,

the error  $\Pi_v$  is within the error of the direct problem. The results of the numerical experiments show that the error of the determination of combustion front velocity is only slightly greater than the error of the initial data - particularly in regard to the model parameters  $\rho c$ ,  $\lambda$ ,  $z_{in}$ ,  $z_{ex}$ ,  $T$ ,  $\alpha$  - and that it increases somewhat with an increase in  $v$ .

#### NOTATION

$s$ , width of the combustion front;  $v$ , velocity of the combustion front;  $\tau$ , time;  $c$ , heat capacity;  $\rho$ , density;  $\lambda$ , thermal conductivity;  $T$ , temperature on the combustion front in the reservoir;  $\tau_{max}$ , time of attainment of the maximum temperature at a given point of the region;  $\Pi_\tau$ , error of measurement of  $\tau_{max}$ ;  $\Pi_v$ , error of determination of velocity  $v$ ;  $Pe$ , Peclet number;  $Nu$ , Nusselt number;  $Fo$ , Fourier number.

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#### OPTIMUM PLANNING OF EXPERIMENTS IN THE IDENTIFICATION OF HEAT-TRANSFER PROCESSES

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UDC 519.24

An analysis is made of problems involving the optimum planning of nonsteady-state experiments conducted to identify thermal processes in structural materials and elements.

In mathematical models used for the theoretical analysis of the thermal operating conditions of different materials and structures, it is possible to distinguish three interconnected parts: 1) internal heat transfer; 2) heat transfer on the surface interacting with the environment; 3) applied thermal loads. Each of these components of the overall model is usually written approximately with allowance for the main governing factors, and each usually contains several characteristics. Identification methods, based on the solution of inverse heat-conduction problems, have recently begun to be widely used to determine these characteristics.

As an example, we will examine a unidimensional process involving the unilateral heating of a structural element with allowance for radiation from the heated surface. The mathematical model of the process has the form

$$C(T) \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x} \left( \lambda(T) \frac{\partial T}{\partial x} \right) + S(T), \quad 0 < x < b, \quad 0 < \tau \leq \tau_m, \quad (1)$$

$$T(x, 0) = T_0(x), \quad 0 \leq x \leq b, \quad (2)$$

$$\frac{\partial T(0, \tau)}{\partial x} = 0, \quad (3)$$

$$q_\lambda(\tau) = -\lambda(T(b, \tau)) \frac{\partial T(b, \tau)}{\partial x} = q(\tau) - \epsilon(T) \sigma T_w^4. \quad (4)$$

Equation (1) describes internal heat transfer in the material of the structural element and contains the characteristics  $C(T)$ ,  $\lambda(T)$ , and  $S(T)$ . Heat balance equation (4) establishes the model of heat transfer on the surface of the structure which interacts with the environment, and it includes the characteristic  $\epsilon(T)$  and the thermal load  $q(\tau)$ . Here, the value of  $q(\tau)$  can be determined by calculation [2]. The characteristics  $C(T)$ ,  $\lambda(T)$ ,